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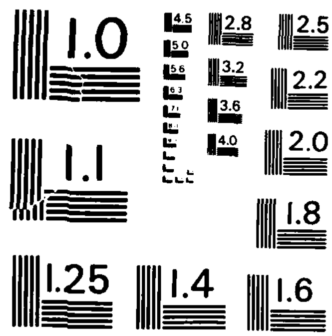

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CHARACTERIZATION OF THE CONSISTENT WEIGHTED REGRESSION ESTIMATORS

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ABSTRACT

The ratio estimator can be justified by a linear superpopulation model without intercept and the error variance proportional to the size of the covariate. If either of the assumptions is violated, then other estimators may be considered. We study the consistency of several estimators which are based on different assumptions about variance structure of the error. Some decompositions of the finite population are introduced. Roughly speaking, we fit a weighted regression line to the finite population with the weight chosen according to the estimator under consideration. We show that any estimators in that class, except the ratio estimator, are inconsistent unless some conditions on the population's characteristics are satisfied. Based on the decomposition, modifications can be made to get a consistent estimator. For the case of p-auxiliary variables, we characterize the class of consistent weighted least squares estimators. The result is extended to the infinite population problem using a completely different approach.

AMS (MOS) Subject Classification: 62D05

Key Words: Weighted regression estimator; Consistency; Ratio estimator; Finite population decomposition.

Work Unit Number 4 - Statistics and Probability

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SIGNIFICANCE AND EXPLANATION

In estimating the population mean of a character of interest, we often make use of an auxiliary covariate which is already available. Ratio estimator is one of the commonly used estimators in survey sampling. The ratio estimator can be justified by a linear superpopulation model with certain error structure assumptions. If some of the assumptions are violated, then other estimators may be considered. We study the consistency of several estimators which are based on different assumptions about variance structure of the error. Some decompositions of the finite population are introduced. We show that the ratio estimator is the only consistent estimator in the class. Based on the decomposition, modifications for other estimators can be made to be consistent estimators. For the case of p-auxiliary variables, we show how to choose the right weight for the weighted regression estimators to be consistent. The result is extended to the infinite population situation.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

CHARACTERIZATION OF THE CONSISTENT WEIGHTED REGRESSION ESTIMATORS

Lih-Yuan Deng*

1. Introduction

Consider a finite population consisting of N units with values (y_i, x_i), where x_i is positive and known for $1 \le i \le N$. A simple random sample of size n is chosen without replacement from the population. Denote the sample and population means of y and x by $\bar{y}, \bar{x}$ and $\bar{Y}, \bar{X}$ respectively. The ratio estimator

$$\hat{\bar{y}}_R = \frac{\bar{y}}{\bar{x}} \bar{X}$$

is the most commonly used estimator of $\bar{Y}$. It is the best linear unbiased predictor of $\bar{Y}$ under the following superpopulation model (Brewer, 1963; Royall, 1970)

$$y_i = \beta x_i + \varepsilon_i \tag{1.1}$$

$$E_M(\varepsilon_i) = 0 \;;\; E_M(\varepsilon_i \varepsilon_j) = \begin{cases} \sigma^2 w_i & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

with $w_i = x_i$, where E_M denotes expectation with respect to the superpopulation model (1.1).

If the variance of ε_i is not proportional to x_i, one may consider the best linear unbiased estimator (BLUE) of $\bar{Y}$

$$\hat{\bar{y}}_w = \frac{\sum_{i=1}^{n} \frac{x_i y_i}{w_i}}{\sum_{i=1}^{n} \frac{x_i^2}{w_i}} \bar{X} = \hat{\beta}_w \bar{X}, \tag{1.2}$$

where $\hat{\beta}_w$ is the weighted least square estimator of β in (1.1). In particular, for w_i =

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1, we have

$$\hat{\bar{y}}_c = \frac{\sum_{i=1}^{n} x_i y_i}{\sum_{i=1}^{n} x_i^2} \bar{X} . \tag{1.3}$$

For $w_i = x_i$, we have $\hat{\bar{y}}_R$. For $w_i = x_i^2$, we have

$$\hat{\bar{y}}_h = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{x_i} \bar{X} . \tag{1.4}$$

Royall(1970) took a different approach to the estimating problem for $\bar{Y}$. He showed that

$$\hat{y} = f\,\bar{y} + (1-f)\,\hat{\beta}_w\ \bar{x}_r \tag{1.5}$$

is the best linear unbiased **predictor** (in contrast to estimator) of $\bar{Y}$ under superpopulation model (1.1), where $f = n/N$ is the sampling fraction, and

$$\bar{x}_r = (N\,\bar{X} - n\,\bar{x})/(N - n) .$$

It is easy to see the difference between the two is

$$\hat{y} - \hat{\bar{y}}_w = f(\,\bar{y} - \hat{\beta}_w\,\bar{x}) . \tag{1.6}$$

From (1.6) one can easily see that

$$\hat{y} - \hat{\bar{y}}_w \longrightarrow 0 \text{ as } n \longrightarrow \infty .$$

either model (1.1) is true or $f \longrightarrow 0$ as $n \longrightarrow \infty$. In practice, the sampling fraction is indeed very small. In either case, this implies $\hat{y}$ and $\hat{\bar{y}}_w$ are asymptotically equivalent. The same remark can be applied for $\hat{\bar{y}}_c$ and $\hat{\bar{y}}_h$.

It is well-known that $\hat{\bar{y}}_R$ is a consistent estimator of $\bar{Y}$. We will first study the asymptotic properties of $\hat{\bar{y}}_c$, $\hat{\bar{y}}_h$ in Sections 1 and 2. More general results for $\hat{\bar{y}}_w$

are obtained in Section 3. In fact, we can show that $\hat{\bar{y}}_R(w_i = x_i)$ is the only consistent estimator of $\bar{Y}$ among $\hat{\bar{y}}_w$. We also show that $\hat{\bar{y}}_c$, $\hat{\bar{y}}_h$ can be consistent estimators of $\bar{Y}$, provided that some conditions on the population characteristics are satisfied. Based on these, we can easily modify $\hat{\bar{y}}_c$, $\hat{\bar{y}}_h$ to get consistent estimators of $\bar{Y}$. A useful decomposition of the finite population is developed. In Section 3, we extend the results in Section 2 to the case of p-dimensional auxiliary variables. Lastly, similar properties can be found for the case of infinite population in Section 4. The techniques used are quite different from those in the previous sections.

The asymptotic behavior of an estimator will be studied under the traditional randomization distribution generated by repeated sampling form a fixed population according to simple random sampling (s.r.s). Following the usual formulation of asymptotic theory in finite population (see for example, Scott and Wu, 1981; Isaki and Fuller, 1982), we embed our finite population in a sequence of populations $\{ U_v \}$ indexed by v with population size N_v and a a simple random sample of size n_v, where $0 < N_1 < N_2 < \ldots$ and $U_1 \subset U_2 \subset U_3 \cdots$. Let a sequence of samples $\{ s_v \}$ of size $\{ n_v \}$ be created from the sequence of populations $\{ U_v \}$ by a sequence of designs, where $n_1 < n_2 < \ldots$ and $n_v < N_v$ for all v. Note that while the sequence of populations is nested, the sequence of samples is not.

Let the population U_v consist of N_v units with values $\{ (y_i, x_i), i=1,\ldots, N_v \}$. The population mean $\bar{X}_v$ of the auxiliary variates is assumed known. $\bar{Y}_v$ denotes the population mean of the character under study. Let $\bar{x}_v, \bar{y}_v$ denote the sample means of x and y based on a s.r.s. without replacement (srswo) of size n_v drawn from popu-

lation U_v.

For a general sampling scheme, Isaki and Fuller(1983) gave a sufficient conditions for the Horvitz-Thompson estimator to be consistent. For the simple random sampling, Lemma 1.1 below is a corollary of Lemma 1 of Isaki and Fuller(1983).

Lemma 1.1. Let $\{ z_i \}$ be a fixed sequence, and satisfies the condition

$$\frac{1}{N_v} \sum_{i=1}^{N_v} (z_i - \overline{Z}_v)^2 < M < \infty \text{ for all } v . \tag{1.5}$$

Then

$$\overline{z}_v - \overline{Z}_v = O_p(n_v^{-0.5}) , \text{ where } \overline{z}_v = \frac{1}{n_v} \sum_{i=1}^{n_v} z_i \text{ and } \overline{Z}_v = \frac{1}{N_v} \sum_{i=1}^{N_v} z_i . \tag{1.6}$$

Throughout this paper, we will assume that for any "reasonable" function g , $z_i = g(x_i, y_i. w_i)$ will satisfy condition (1.5).

2. Asymptotic Behavior of $\hat{\overline{y}}_c$, $\hat{\overline{y}}_h$ and $\hat{\overline{y}}_w$

2.1. Asymptotic Behavior of $\hat{\overline{y}}_c$

We can fit the regression line to the finite population $\{ (y_i, x_i),i=1,2,.., N_v \}$

$$y_i = \alpha_{c,v} + \beta_{c,v} x_i + d_i , \tag{2.1}$$

where

$$\beta_{c,v} = \frac{\sum_{i=1}^{N_v} (x_i - \overline{X}_v)(y_i - \overline{Y}_v)}{\sum_{i=1}^{N_v} (x_i - \overline{X}_v)^2} \tag{2.2}$$

and

$$\alpha_{c,v} = \overline{Y}_v - \beta_{c,v}\,\overline{X}_v\,, \tag{2.3}$$

$$d_i = y_i - \alpha_{c,v} - \beta_{c,v}\,x_i\,. \tag{2.4}$$

It is straightforward to see $\{\ d_i\ \}$ satisfy the following

$$\sum_{i=1}^{N_v} d_i = 0\,, \sum_{i=1}^{N_v} d_i\,x_i = 0\,. \tag{2.5}$$

We can use the decomposition in (2.1) to characterize the asymptotic behavior of

$$\hat{\overline{y}}_{c,v} = \frac{\sum_{i=1}^{n_v} x_i\,y_i}{\sum_{i=1}^{n_v} x_i^2}\,\overline{X}_v\,. \tag{2.6}$$

Theorem 2.1.

(a) $\hat{\overline{y}}_{c,v} - \overline{Y}_v = \alpha_{c,v}[\,-\dfrac{S^2_{x,v}}{\overline{X}_v^{(2)}}] + O_p(\,n_v^{-0.5})\,.$

(b) $\hat{\overline{y}}_{c,v} - \overline{Y}_v \xrightarrow{p} 0$, if $\lim\limits_{v \to \infty} \alpha_{c,v} = 0\,,$

where

$$S^2_{x,v} = \frac{1}{N_v - 1}\sum_{i=1}^{N_v}(\,x_i - \overline{X}_v)^2\,,\ \overline{X}_v^{(2)} = \frac{1}{N_v}\sum_{i=1}^{N_v} x_i^2\,.$$

Proof. Using (2.1) and (2.6), we have

$$\hat{\overline{y}}_{c,v} = \alpha_{c,v}\frac{\overline{x}_v}{\overline{x}_v^{(2)}}\,\overline{X}_v + \beta_{c,v}\,\overline{X}_v + \frac{\frac{1}{n_v}\sum_{i=1}^{n_v} x_i\,d_i}{\overline{x}_v^{(2)}}\,\overline{X}_v\,,\ \text{where}\ \ \overline{x}_v^{(2)} = \frac{1}{n_v}\sum_{i=1}^{n_v} x_i^2\,.$$

This together with $\overline{Y}_v = \alpha_{c,v} + \beta_{c,v}\,\overline{X}_v$, implies

$$\hat{\overline{y}}_{c,v} - \overline{Y}_v = \alpha_{c,v}[\frac{\overline{x}_v}{\overline{x}_v^{(2)}}\,\overline{X}_v - 1] + \frac{\frac{1}{n_v}\sum_{i=1}^{n_v} x_i\,d_i}{\overline{x}_v^{(2)}}\,\overline{X}_v. \tag{2.7}$$

From (2.7), $\dfrac{1}{n_v}\sum_{i=1}^{n_v} x_i\,d_i = O_p(\,n_v^{-0.5})$. and $\dfrac{\overline{x}_v}{\overline{x}_v^{(2)}}\,\overline{X}_v = \dfrac{\overline{X}_v^{\,2}}{\overline{X}_v^{(2)}} + O_p(\,n_v^{-0.5})$, we

have Part(a). Part(b) follows from Part(a) and $\frac{S_{x,v}^2}{\overline{X}_v^{(2)}} \leq 1$. □

From Part(a) of Theorem 1, the leading term of $\hat{\overline{y}}_{c,v} - \overline{Y}_v$ depends only on the sign of $\alpha_{c,v}$, the intercept of the regression line. If $\alpha_{c,v} > 0$ (< 0), then $\hat{\overline{y}}_{c,v}$ has a positive (negative) bias of constant order. If $\alpha_{c,v} \longrightarrow 0$ as $v \longrightarrow 0$, then $\hat{\overline{y}}_{c,v}$ is a consistent estimator of $\overline{Y}_v$. In practice, $\alpha_{c,v}$ is of course unknown. However, we can estimate $\alpha_{c,v}$ consistently by using the sample analog $\hat{\alpha}_{c,v}$ of $\alpha_{c,v}$ in (2.1). It is easy to modify $\hat{\overline{y}}_{c,v}$ to get a consistent estimator of $\overline{Y}_v$. For example, $\hat{\overline{y}}_{c,v} + \hat{\alpha}_{c,v}\frac{S_{x,v}^2}{\overline{X}_v^{(2)}}$ and $\hat{\overline{y}}_{c,v} + \hat{\alpha}_{c,v}\frac{s_{x,v}^2}{\overline{x}_v^{(2)}}$ are consistent estimators of $\overline{Y}_v$, where $s_{x,v}^2, \overline{x}_v^{(2)}$ are the sample analogs of $S_{x,v}^2, \overline{X}_v^{(2)}$.

2.2. Asymptotic Behavior of $\hat{\overline{y}}_{h,v}$

We need a different decomposition of the population for studying the asymptotic behavior of

$$\hat{\overline{y}}_{h,v} = \frac{1}{n_v} \left(\sum_{i=1}^{n_v} \frac{y_i}{x_i}\right) \overline{X}_v . \tag{2.8}$$

Fit a weighted regression line with weight proportional to x_i^{-1} to the finite population U_v

$$y_i = \alpha_{h,v} + \beta_{h,v}\, x_i + d_i , \tag{2.9}$$

where

$$\beta_{h,v} = \frac{\overline{Y}_v \, \overline{X}_v^{(-1)} - \frac{1}{N_v} \sum_{i=1}^{N_v} \frac{y_i}{x_i}}{\overline{X}_v \, \overline{X}_v^{(-1)} - 1}, \tag{2.10}$$

$$\alpha_{h,v} = \frac{\overline{X}_v \frac{1}{N_v} \sum_{i=1}^{N_v} \frac{y_i}{x_i} - \overline{Y}_v}{\overline{X}_v \, \overline{X}_v^{(-1)} - 1}, \tag{2.11}$$

$$d_i = y_i - \alpha_{h,v} - \beta_{h,v} \, x_i \, .$$

Note that $\{ \, d_i \, \}$ satisfy the following

$$\sum_{i=1}^{N_v} d_i = 0 \, , \sum_{i=1}^{N_v} \frac{d_i}{x_i} = 0 \, . \tag{2.12}$$

Using the above decomposition, we can find the leading term of $\hat{\overline{y}}_{h,v} - \overline{Y}_v$.

Theorem 2.2.

(a) $\hat{\overline{y}}_{h,v} - \overline{Y}_v = \alpha_{h,v}[\, \overline{X}_v \, \overline{X}_v^{(-1)} - 1] + O_p(\, n_v^{-0.5}) \, .$

(b)If $\lim_{v \to \infty} \overline{X}_v^{(-1)} < \infty$, then $\hat{\overline{y}}_{h,v} - \overline{Y}_v \xrightarrow{p} 0$, if $\alpha_{h,v} \longrightarrow 0$ as $v \longrightarrow \infty$,

where $\overline{X}_v^{(-1)} = \frac{1}{N_v} \sum_{i=1}^{N_v} x_i^{-1}$.

Proof. From (2.8) and (2.9), we have

$$\hat{\overline{y}}_{h,v} = \alpha_{h,v} \frac{1}{n_v} \sum_{i=1}^{n_v} \frac{1}{x_i} \, \overline{X}_v + \beta_{h,v} \, \overline{X}_v + \left(\frac{1}{n_v} \sum_{i=1}^{n_v} \frac{d_i}{x_i}\right) \overline{X}_v \, .$$

This together with $\overline{Y}_v = \alpha_{h,v} + \beta_{h,v} \, \overline{X}_v$ implies

$$\hat{\overline{y}}_{h,v} - \overline{Y}_v = \alpha_{h,v}(\, \overline{x}_v^{(-1)} \, \overline{X}_v - 1) + \frac{1}{n_v} \sum_{i=1}^{n_v} \frac{d_i}{x_i} \, \overline{X}_v \, .$$

Part(a) follows from the above expression and $\overline{x}_v^{(-1)} = \overline{X}_v^{(-1)} + O_p(\, n_v^{-0.5})$,

$\frac{1}{n_v} \sum_{i=1}^{n_v} \frac{d_i}{x_i} = O_p(\, n_v^{-0.5})$. Part(b) follows immediately from Part(a). □

From the Jensen's inequality, we can see that $\overline{X}_v \overline{X}_v^{(-1)} - 1 > 0$. Therefore, the sign of the leading term of $\hat{\overline{y}}_{h,v} - \overline{Y}_v$ depends only on $\alpha_{h,v}$, the intercept of the weighted regression line. If $\alpha_{h,v} > 0$ (<0), then $\hat{\overline{y}}_{h,v}$ has negative (positive) bias of constant order. Theorem 2.2 not only gives a condition for $\hat{\overline{y}}_{h,v}$ to be consistent but also indicates how to modify $\hat{\overline{y}}_{h,v}$ to get consistent estimators of $\overline{Y}_v$.

2.3. Asymptotic Behavior of $\hat{\overline{y}}_{w,v}$

To study the asymptotic behavior of

$$\hat{\overline{y}}_{w,v} = \frac{\sum_{i=1}^{n_v} \frac{x_i y_i}{w_i}}{\sum_{i=1}^{n_v} \frac{x_i^2}{w_i}} \overline{X}_v , \tag{2.13}$$

we need the following decomposition of the finite population

$$y_i = \alpha_{w,v} w_i + \beta_{w,v} x_i + d_i , \tag{2.14}$$

where d_i satisfy the following

$$\sum_{i=1}^{N_v} d_i = 0 , \sum_{i=1}^{N_v} \frac{x_i d_i}{w_i} = 0 . \tag{2.15}$$

Note that if the vector of $\{ w_i\}$ and the vector of $\{ x_i\}$ are linearly independent, then $\alpha_{w,v}$, $\beta_{w,v}$ will be uniquely determined under condition (2.15). We can obtain $\alpha_{w,v}$, $\beta_{w,v}$ by fitting the weighted least square line as in (2.14), with the weight proportional to w_i. Using the above decomposition we can show the following theorem:

Theorem 2.3. If $\alpha_{w,v}$, $\beta_{w,v}$ can be uniquely determined in (2.14), then

(a) $\hat{\bar{y}}_{w,v} - \bar{Y}_v = \alpha_{w,v}\left[\dfrac{\bar{X}_v^{\,2}}{N_v^{-1}\sum_{i=1}^{N_v} x_i^2 w_i^{-1}} - \bar{W}_v\right] + O_p(n_v^{-0.5})$.

(b) $\left[\dfrac{\bar{X}_v^{\,2}}{N_v^{-1}\sum_{i=1}^{N_v} x_i^2 w_i^{-1}} - \bar{W}_v\right] \le 0$, for all N_v.

(c) If $\lim_{v \to \infty} \bar{W}_v < \infty$ and $0 < \lim_{v \to \infty} \dfrac{1}{N_v}\sum_{i=1}^{N_v} x_i^2 w_i^{-1} < \infty$,

then $\hat{\bar{y}}_{w,v} - \bar{Y}_v \xrightarrow{p} 0$, if $\alpha_{w,v} \longrightarrow 0$ as $v \longrightarrow \infty$,

where $\bar{W}_v = \dfrac{1}{N_v}\sum_{i=1}^{N_v} w_i$.

Proof. From (2.13) and (2.14), we have

$$\hat{\bar{y}}_{w,v} = \alpha_{w,v}\frac{\bar{x}_v}{n_v^{-1}\sum_{i=1}^{n_v} w_i^{-1} x_i^2} + \beta_{w,v}\bar{X}_v + \frac{n_v^{-1}\sum_{i=1}^{n_v} w_i^{-1} x_i d_i}{n_v^{-1}\sum_{i=1}^{n_v} w_i^{-1} x_i^2}.$$

From this and $\bar{Y}_v = \alpha_{w,v}\bar{W}_v + \beta_{w,v}\bar{X}_v$, we have

$$\hat{\bar{y}}_{w,v} - \bar{Y}_v = \alpha_{w,v}\left[\frac{\bar{x}_v}{n_v^{-1}\sum_{i=1}^{n_v} w_i^{-1} x_i^2}\bar{X}_v - \bar{W}_v\right] + \frac{n_v^{-1}\sum_{i=1}^{n_v} w_i^{-1} x_i d_i}{n_v^{-1}\sum_{i=1}^{n_v} w_i^{-1} x_i^2}.$$

Part(a) follows from the above expression and

$$\frac{1}{n_v}\sum_{i=1}^{n_v}\frac{x_i d_i}{w_i} = O_p(n_v^{-0.5})$$

Part(b) follows from the Cauchy-Schwartz inequality. Part(c) follows immediately from Part(a). □

From Theorem 2.3, we can see that the leading term of $\hat{\bar{y}}_{w,v}$ - $\overline{Y}_v$ depends only on $\alpha_{w,v}$, the "intercept" of the regression "line" in (2.14). Moreover, If $\alpha_{w,v} > 0$ (< 0) , then $\hat{\bar{y}}_{w,v}$ has negative (positive) bias of constant order. Theorems 2.1 and 2.2 are special cases of Theorem 2.3 with $w_i = 1$ and $w_i = x_i^2$. However, Theorems 2.1 and 2.2 give the explicit formulae for the leading term bias of $\hat{\bar{y}}_c$ and $\hat{\bar{y}}_h$ in terms of the usual regression decomposition of the population.

3. p-dimensional Auxiliary Variables

In Section 2, we assume that only one auxiliary variable (x) is available. Estimators like $\hat{\bar{y}}_{w,v} = \hat{\beta}_{w,v}\,\overline{X}_v$ are justified by the superpopulation model (1.1). The regression estimator is another popular estimator of $\overline{Y}_v$. It is the best linear unbiased predictor under the following superpopulation model (Royall, 1970)

$$y_i = \alpha + \beta\, x_i + \varepsilon_i \tag{3.1}$$

$$E_M(\varepsilon_i) = 0 \;;\; E_M(\varepsilon_i\,\varepsilon_j) = \begin{cases} \sigma^2 w_i & \text{if } i=j \\ 0 & \text{if } i \neq j \end{cases}$$

with $w_i = 1$.

If the constant variance assumption is in question, one may like to consider an estimator of $\overline{Y}_v$ which is based on the weighted least estimator of α,β with some weight w_i. As we have seen in Section 2, not all kinds of weight will give us consistent estimators. As a matter of fact, in Section 2 the only weight we may choose such that $\hat{\bar{y}}_{w,v}$ is consistent is $w_i = x_i$. We may ask the same question for the weighted regression estimators. More generally, for more than one auxiliary variates, the commonly used

estimator is the multiple regression estimator. We may again consider a weighted least square estimator for p-auxiliary variables. We would like to know what kind of weight will result in a consistent estimator. If the weight does not give consistent estimator for all populations, we would like to characterize the leading term of the bias in terms of some simple population characteristics so that we may modify the estimator to get a consistent estimator. We would like to extend the results in Section 2 to the case of several auxiliary variables. Some notations will be introduced.

Consider a finite population U_v indexed by v with population size N_v and a simple random sample (s.r.s.) with size n_v. Let $X_1, X_2, .., X_p$ be the p-auxiliary variates which are correlated with character Y. The purpose is again to estimate $\overline{Y}_v$, the population mean of the variable of interest. Under simple random sampling, a sample (y_{s_v}, X_{s_v}) of size n_v is chosen, where $y_{s_v} = (y_{i_1}, y_{i_2}, \cdots, y_{i_{n_v}})'$, $X_{s_v} = (\underset{\sim}{X}_{i_1}, \underset{\sim}{X}_{i_2}, \cdots, \underset{\sim}{X}_{i_{n_v}})'$, $\underset{\sim}{X}_i = (x_{1i}, x_{2i}, \cdots, x_{pi})'$.

Consider a general class of weighted least square estimators

$$\hat{\overline{y}}_{w,v} = \overline{X}_v (X_{s_v}' W_{s_v}^{-1} X_{s_v})^{-1} X_{s_v}' W_{s_v}^{-1} y_{s_v} = \overline{X}_v \hat{\beta}_{w,v} \tag{3.2}$$

where

$$W_{s_v} = \mathrm{diag}(w_{i_1}, w_{i_2}, \cdots w_{i_{n_v}}), \quad \overline{X}_v = \frac{1}{N_v} \sum_{i=1}^{N_v} \underset{\sim}{X}_i .$$

and $w_i > 0$ is the weight associated with unit i.

Note that ratio estimator is a special case of (3.2), with p=1, $w_i = x_i$, and the regression estimator estimator is also a special case of (3.2), with p=2, $w_i = 1$, and the first column of $\underset{\sim}{X}$ matrix is all 1 's and the second column is X_i's.

An important question one may ask is that what kind of weight W_i we can choose such that $\hat{\bar{y}}_{w,v}$ will be a consistent estimator for $\bar{Y}_v$? If $\hat{\bar{y}}_{w,v}$ is not consistent we would like to find the leading term of $\hat{\bar{y}}_{w,v}$ - $\bar{Y}_v$ in terms of some simple population characteristics. Hence, one can modify $\hat{\bar{y}}_{w,v}$ to get a consistent estimator of $\bar{Y}_v$.

To characterize the asymptotic behavior of $\hat{\bar{y}}_{w,v}$, we need the following decomposition of the finite population U_v

$$y_v = X_v \beta_v + Z_v \gamma_v + d_v \tag{3.3}$$

where

$$Z_v = (w_1, w_2, \ldots, w_{N_v})' , W_v = \mathrm{diag}(w_1, w_2, \ldots, w_{N_v})' \tag{3.4}$$

and

$$y_v = (y_1, y_2, \ldots, y_{N_v})' , d_v = (d_1, d_2, \ldots, d_{N_v})' ,$$

$$X_v = (\underset{\sim}{X}_1, \underset{\sim}{X}_2, \ldots, \underset{\sim}{X}_{N_v})' , \underset{\sim}{X}_i = (x_{1i}, x_{2i}, .. x_{pi})' .$$

Note that β_v and γ_v will be uniquely determined, if Z_v is not in the column space of X_v , the dimension of the column space of X_v is p , and

$$X_v' W_v^{-1} d_v = 0 \tag{3.5}$$

and

$$Z_v' W_v^{-1} d_v = 1_{N_v}' d_v = \sum_{i=1}^{N_v} d_i = 0 . \tag{3.6}$$

Here is our key Theorem.

Theorem 3.1. Assume that $(X_v' W_v^{-1} X_v)$ is non-singular, for all v.

(a) If $1_{N_v} \in \text{col}(W_v^{-1} X_v)$, then

$$\hat{\bar{y}}_{w,v} - \bar{Y}_v = (\bar{y}_v - \bar{Y}_v) - (\bar{x}_v - \bar{X}_v) \beta_{w,v} + O_p(n_v^{-1})$$

where $1_{N_v} = (1, 1,.., 1)'$. $\hat{\bar{y}}_{w,v}$ is then obviously a consistent estimator of $\bar{Y}_v$.

(b) If $1_{N_v} \notin \text{col}(W_v^{-1} X_v)$, then

$$\hat{\bar{y}}_{w,v} - \bar{Y}_v = [\bar{X}_v(\frac{1}{N_v} X_v' W_v^{-1} X_v)^{-1} \bar{X}_v' - \bar{W}_v] \gamma_v + O_p(n_v^{-0.5}) , \quad (3.7)$$

where

$$[\bar{X}_v(\frac{1}{N_v} X_v' W_v^{-1} X_v)^{-1} \bar{X}_v' - \bar{W}_v] \leq 0 , \text{ for all } v . \quad (3.8)$$

$\hat{\bar{y}}_{w,v} - \bar{Y}_v \longrightarrow 0$, if $\gamma_v \longrightarrow 0$ as $v \longrightarrow \infty$.

Proof. Note that

$$\hat{\bar{y}}_{w,v} = \bar{x}_v \hat{\beta}_{w,v} - (\bar{x}_v - \bar{X}_v) \hat{\beta}_{w,v} . \quad (3.9)$$

The assumption of Part(a) implies $1_{n_v}' = c' X_{s_v}' W_{s_v}^{-1}$ for some c , from which, we have

$$\bar{x}_v \hat{\beta}_{w,v} = \frac{1}{n_v} c' X_{s_v}' W_{s_v}^{-1} y_{s_v} = \bar{y}_v . \quad (3.10)$$

Note that

$$\frac{1}{n_v} (X_{s_v}' W_{s_v}^{-1} X_{s_v}) = \frac{1}{N_v} (X_v' W_v^{-1} X_v) + O_p(n_v^{-0.5}) \quad (3.11)$$

and

$$\frac{1}{n_v} X_{s_v}{}' W_{s_v}{}^{-1} y_{s_v} = \frac{1}{N_v} X_v{}' W_v{}^{-1} y_v + O_p(n_v^{-0.5}) .$$

Since $(X_v{}' W_v{}^{-1} X_v)$ is assumed non-singular for all v

$$[\frac{1}{n_v} (X_{s_v}{}' W_{s_v}{}^{-1} X_{s_v})]^{-1} = [\frac{1}{N_v} (X_v{}' W_v{}^{-1} X_v)]^{-1} + O_p(n_v^{-0.5}) ,$$

which implies

$$\hat{\beta}_{w,v} = \beta_{w,v} + O_p(n_v^{-0.5}) . \tag{3.12}$$

Part(a) follows from (3.10) and (3.12). To prove Part(b), we will use the decomposition (3.3). Note that $1_{N_v} \notin \text{col}(W_v{}^{-1} X_v)$ implies $Z_v = W_v 1_{N_v} \notin \text{col}(X_v)$. Therefore, the coefficients β_v , γ_v in the decomposition (3.3) will be uniquely determined. Using (3.2),(3.3) and $W_{s_v}{}^{-1} Z_{s_v} = 1_{n_v}$, we have

$$\hat{\bar{y}}_{w,v} = \overline{X}_v \beta_v + \overline{X}_v(X_{s_v}{}' W_{s_v}{}^{-1} X_{s_v})^{-1} n_v \bar{x}_v \gamma_v$$
$$+ \overline{X}_v(X_{s_v}{}' W_{s_v}{}^{-1} X_{s_v})^{-1} X_{s_v}{}' W_{s_v}{}^{-1} d_{s_v} .$$

From (3.3) and (3.6), it is straightforward to see

$$\overline{Y}_v = \overline{X}_v \beta_v + \overline{Z}_v \gamma_v + 0 = \overline{X}_v \beta_v + \overline{W}_v \gamma_v .$$

Taking the difference, we get

$$\hat{\bar{y}}_{w,v} - \overline{Y}_v = [\overline{X}_v(\frac{1}{n_v} X_{s_v}{}' W_{s_v}{}^{-1} X_{s_v})^{-1} \bar{x}_v - \overline{W}_v] \gamma_v$$
$$+ \overline{X}_v(\frac{1}{n_v} X_{s_v}{}' W_{s_v}{}^{-1} X_{s_v})^{-1} (\frac{1}{n_v} X_{s_v}{}' W_{s_v}{}^{-1} d_{s_v}) \tag{3.13}$$

Using (3.11) and

$$\frac{1}{n_v} X_{s_v}{}' W_{s_v}{}^{-1} d_{s_v} = \frac{1}{N_v} X_v{}' W_v{}^{-1} d_v + O_p(n_v^{-0.5}) = O_p(n_v^{-0.5}) ,$$

it is easy to see last term of (3.13) is $O_p(n_v^{-0.5})$. (3.7) follows from (3.13) and $\bar{x}_v - \overline{X}_v = O_p(n^{-0.5})$. And (3.8) holds because

$$\overline{W}_v - \overline{X}_v\left(\frac{1}{N_v} X_v' W_v^{-1} X_v\right)^{-1} \overline{X}_v'$$

$$= \frac{1}{N_v} 1_{N_v}'(W_v - X_v (X_v' W_v^{-1} X_v)^{-1} X_v')1_{N_v}$$

$$= \frac{1}{N_v} [c' (I - M(M'M)^{-1}M') c] \geq 0 ,$$

where $M = W_v^{-1/2} X_v$, $c = W_v^{1/2} 1_{N_v}$ and $I - M(M'M)^{-1}M'$ is a positive semi-definite matrix. □

Wright(1983) considered a class of asymptotically design-unbiased (ADU) predictors under a general sampling design, our conclusion in Part(a) is the same as his Theorem 1. However, Part(b) of our Theorem 3.1 uses some simple population characteristics to characterize the leading term of $\hat{\overline{y}}_{w,v} - \overline{Y}_v$ when 1_{N_v} is not in the column space of $W_v^{-1} X_v$. From Part(b), we can then easily construct some new consistent estimators of $\overline{Y}_v$. For example, let $\hat{\gamma}_v$ be the sample analog of γ_v , then $\hat{\overline{y}}_{w,v} - \overline{X}_v[(\frac{1}{n_v} X_v' W_v^{-1} X_v)^{-1} \overline{X}_v - \overline{W}_v] \hat{\gamma}_v$ is a consistent estimator of $\overline{Y}_v$.

4. Characterization of a Class of Consistent Estimators in Infinite Population

To characterize a class of consistent estimators for infinite populations, the techniques used in Section 2 for finite population are not appropriate. A completely different approach will be taken. The asymptotic framework for the infinite population is the standard one. Therefore, the notation is simpler.

Let ($Y_i , \underset{\sim}{X}_i$) be an i.i.d random vectors with distribution $F(Y, \underset{\sim}{X})$, i=1,2, .. , n, where Y_i is the variable of interest and $\underset{\sim}{X}_i$ is a p-dimensional variables. The joint distribution F of Y and $\underset{\sim}{X}$ need not to be known. The main purpose is to use both $\underset{\sim}{X}$

and Y to estimate $E(Y) = \mu_y$, where E denotes the expectation.

Consider a class of estimators

$$\hat{\mu}_y = \hat{\underset{\sim}{\mu}}_x{}'(A'V^{-1}A)^{-1}A'V^{-1}\underset{\sim}{Y}_s = \hat{\underset{\sim}{\mu}}_x{}' \, \hat{\underset{\sim}{\beta}}_w , \tag{4.1}$$

where $\hat{\underset{\sim}{\mu}}_x$ is any consistent estimator of $E(\underset{\sim}{X})$, i.e.

$$\hat{\underset{\sim}{\mu}}_x = E(\underset{\sim}{X}) + o_p(1)$$

and

$$\hat{\underset{\sim}{\beta}}_w = (A'V^{-1}A)^{-1}A'V^{-1}\underset{\sim}{Y}_s \quad , \underset{\sim}{Y}_s = (Y_1, \cdots, Y_n)' ,$$

$$A_{nxp} = (\underset{\sim}{X}_1, \underset{\sim}{X}_2, \cdots, \underset{\sim}{X}_n)' , \underset{\sim}{X}_i = (Z_{1i}, Z_{2i}, .., Z_{pi})' ,$$

$$V = \mathrm{diag}(W_1, W_2, .., W_n) , \; W_i = g(\underset{\sim}{X}_i) \text{ for some function g.}$$

Note that the estimator in (4.1) is appropriate, if

$$Y_i = X_i' \underset{\sim}{\beta} + \varepsilon_i , \tag{4.2}$$

where ε_i are i.i.d. with mean zero and finite variance. Since $\hat{\underset{\sim}{\beta}}_w$ is the weighted least estimator of $\underset{\sim}{\beta}$, it can be shown that $\hat{\underset{\sim}{\beta}}_w$ is a consistent estimator of $\underset{\sim}{\beta}$ under (4.2), for any choice of weight W_i. However, if (4.2) is not the true model, say there should be a Z variable in the model

$$Y_i = \underset{\sim}{X}_i{}' \underset{\sim}{\beta} + \underset{\sim}{Z}_i{}' \underset{\sim}{\gamma} + \varepsilon_i , \tag{4.3}$$

then $\hat{\underset{\sim}{\beta}}_w$ will not be consistent for $\underset{\sim}{\beta}$ under (4.3), unless $\underset{\sim}{X}$ and $\underset{\sim}{Z}$ are "orthogonal".

But $\hat{\mu}_y$ will still be a consistent estimator for $E(Y)$ even if (4.2) is not the true model provided we choose the "right" weight W_i. It is our purpose in this section to

characterize the weight W_i such that $\hat{\mu}_y$ will be consistent for estimating $E(Y)$.

In the context of survey sampling, the $\underset{\sim}{X}$ variable is called the auxiliary variable. Hence, usually the marginal distribution of $\underset{\sim}{X}$ is known, or $E(\underset{\sim}{X})$ is available. In that case, we may choose $\hat{\underset{\sim}{\mu}}_x = E(\underset{\sim}{X})$, and the estimator in (4.1) becomes

$$\hat{\mu}_y = E(\underset{\sim}{X})'(A'V^{-1}A)^{-1}A'V^{-1}\underset{\sim}{Y}_s \quad ,$$

which is similar to $\hat{\bar{y}}_{w,v}$ in Section 3.

Let $\underset{\sim}{X} = (Z_1, Z_2, .., Z_p)'$ where Z_1, the first component of X, has the same distribution as Z_{1i}'s, and Z_2 has the same distribution as Z_{2i}'s, ... etc. Let $W = g(\underset{\sim}{X}) = g((Z_1, Z_2, .., Z_p)')$ be a real random variable. W_i's are i.i.d. with the same distribution as W.

We will impose some mild conditions on the random variables Z_i's, W and Y.

(A0) $\hat{\underset{\sim}{\mu}}_x = E(\underset{\sim}{X}) + o_p(1)$,

(A1) $E\left(\left|\dfrac{Z_i\ Z_j}{W}\right|\right) < \infty$, for all i,j=1,2,..,p ,

(A2) $E\left(\left|\dfrac{Z_j\ Y}{W}\right|\right) < \infty$, for all j=1,2,..,p ,

(A3) $\left[E\left(\left|\dfrac{Z_i\ Z_j}{W}\right|\right)\right]_{i,j} = E(W^{-1}\ \underset{\sim}{X}'\ \underset{\sim}{X})$ is a non–singular matrix.

Before stating our key Theorem, let us derive some properties of $\hat{\mu}_y$ based on these assumptions.

From (A1) and the Weak Law of Large Numbers,

$$\frac{1}{n}(A'V^{-1}A) = \frac{1}{n}\sum_{i=1}^{n} W_i^{-1}\underset{\sim}{X}_i{}'\underset{\sim}{X}_i = E(W^{-1}\underset{\sim}{X}'\underset{\sim}{X}) + o_p(1)\,.$$

Using this and assumption (A3), it is easy to see

$$\left(\frac{1}{n}(A'V^{-1}A)\right)^{-1} = [E(W^{-1}\underset{\sim}{X}'\underset{\sim}{X})]^{-1} + o_p(1)\,. \tag{4.4}$$

From (A2) and the Weak Law of Large Numbers,

$$\frac{1}{n}A'V^{-1}\underset{\sim}{Y}_s = \frac{1}{n}\sum_{i=1}^{n} W_i^{-1}\underset{\sim}{X}_i{}'Y_i = E(W^{-1}\underset{\sim}{X}'Y) + o_p(1)\,. \tag{4.5}$$

Combining (A0), (4.4) and (4.5), we have

$$\hat{\mu}_y = \hat{\underset{\sim}{\mu}}_x{}'\,\hat{\underset{\sim}{\beta}}_w = [\,E(\underset{\sim}{X}) + o_p(1)]'\,[(\frac{1}{n}(A'V^{-1}A))^{-1}(\frac{1}{n}A'V^{-1}\underset{\sim}{Y}_s)]$$

$$= E(\underset{\sim}{X})'[E(W^{-1}\underset{\sim}{X}'\underset{\sim}{X})]^{-1}E(W^{-1}\underset{\sim}{X}'Y) + o_p(1) \tag{4.6}$$

Now we are ready to state our key result.

Theorem 4.1. If assumptions (A0)-(A3) hold, then

$$\hat{\mu}_y = \hat{\underset{\sim}{\mu}}_x(A'V^{-1}A)^{-1}A'V^{-1}\underset{\sim}{Y}_s \xrightarrow{p} E(Y)$$

if and only if

$$W = \sum_{j=1}^{p} c_j Z_j \text{ for some } c_j\text{'s}\,.$$

Proof. (i) Sufficiency: From (4.6)

$$\hat{\mu}_y = E(\underset{\sim}{X})'E(W^{-1}\underset{\sim}{X}'\underset{\sim}{X})^{-1}E(W^{-1}\underset{\sim}{X}'Y) + o_p(1)$$

$$= E(\underset{\sim}{X})'[E(W^{-1}\underset{\sim}{X}'\underset{\sim}{X})^{-1}C^{-1}]\,[C\,E(W^{-1}\underset{\sim}{X}'Y)] + o_p(1)\,, \tag{4.7}$$

where C is any non-singular matrix with the first row equal to $(c_1, c_2, \ldots, c_p)$. Note that

$$C \; E(W^{-1} \underset{\sim}{X}'Y) = \begin{bmatrix} c_1 & c_2 & . & c_p \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} E(W^{-1} Z_1 Y) \\ E(W^{-1} Z_2 Y) \\ \vdots \\ E(W^{-1} Z_p Y) \end{bmatrix} = \begin{bmatrix} E(Y) \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \tag{4.8}$$

and

$$E(W^{-1} \underset{\sim}{X}' \underset{\sim}{X})^{-1} C^{-1} = [C\, E(W^{-1} \underset{\sim}{X}' \underset{\sim}{X})]^{-1} = \begin{bmatrix} E(Z_1) & E(Z_2) & . & E(Z_p) \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}^{-1}, \tag{4.9}$$

which will be denoted by B^{-1}. From (4.7)-(4.9), we get

$$\hat{\mu}_y = [E(Z_1), E(Z_2), .., E(Z_p)]\, B^{-1} \begin{bmatrix} E(Y) \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} + o_p(1)$$

$$= (1,0,0,..,0) \begin{bmatrix} E(Y) \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} + o_p(1) = E(Y) + o_p(1) . \tag{4.10}$$

Therefore, we proved

$$\hat{\mu}_y - E(Y) \xrightarrow{P} 0 .$$

Note (4.10) is true, because

$$[E(Z_1), E(Z_2), .., E(Z_p)] = (1,0,0,..,0)\, B .$$

(ii) Necessity: Assume that as $n \longrightarrow \infty$ $\hat{\mu}_y \xrightarrow{P} E(Y)$. From (4.6), we know that

$$\hat{\mu}_y \xrightarrow{P} E(\underset{\sim}{X})'(E(W^{-1} \underset{\sim}{X}' \underset{\sim}{X}))^{-1} E(W^{-1} X'Y) ,$$

Hence

$$E(Y) = [E(\underset{\sim}{X})'(E(W^{-1} \underset{\sim}{X}' \underset{\sim}{X}))^{-1}] E(W^{-1} X'Y)$$

$$= (d_1, d_2, .., d_p) \begin{bmatrix} E(W^{-1} Z_1 Y) \\ E(W^{-1} Z_2 Y) \\ \vdots \\ E(W^{-1} Z_p Y) \end{bmatrix} = E(\sum_{j=1}^{p} W^{-1} d_j Z_j Y) ,$$

where $(d_1, d_2, \ldots, d_p) = E(\underset{\sim}{X})'[E(W^{-1}\underset{\sim}{X}'\underset{\sim}{X})]^{-1}$. Therefore

$$E([\sum_{j=1}^{p} W^{-1} d_j Z_j - 1]\, Y) = 0, \text{ for any } Y,$$

which implies

$$\sum_{j=1}^{p} W^{-1} d_j Z_j = 1 \text{ with probability } 1.$$

Hence,

$$W = \sum_{j=1}^{p} d_j Z_j .$$

This completes the proof of Theorem 4.1. □

As we can see from the proof of Theorem 4.1, the i.i.d. assumption for $\{(X_i, Y_i)\}$ is not necessary. All we need is formula (4.6) which is implied by (4.4) and (4.5). Note that (4.4) and (4.5) hold for simple random sampling without replacement. Hence, Theorem 4.1 is an extension of Theorem 3.1. One possible application of Theorem 4.1 would be in the missing data problem. Suppose that some of the X-components among the sample are missing, and other data may have only X-component with y missing. In this case, $\hat{\underset{\sim}{\mu}}_x$ may be chosen as a component-wise average of X in the sample.

REFERENCES

Brewer, K. R. W. (1963), "Ratio estimation and finite populations: Some results deducible from the assumption of an underlying stochastic process," *Australian Journal of Statistics*, *5*, 93-105.

Isaki, C. T. and Fuller, W. A. (1982), "Survey design under a regression superpopulation model," *Journal of the American Statistical Association*, *77*, 89-96.

Royall, R. M. (1970), "On finite population sampling theory under certain linear regression models," *Biometrika*, *57*, 377-387.

Scott, A. and Wu, C. F. (1981), "On the asymptotic distribution of ratio and regression estimators," *Journal of the American Statistical Association*, *76*, 98-102.

Wright, R. L. (1983), "Finite population sampling with multivariate auxiliary information," *Journal of the American Statistical Association*, *78*, 879-884.

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The ratio estimator can be justified by a linear superpopulation model without intercept and the error variance proportional to the size of the covariate. If either of the assumptions is violated, then other estimators may be considered. We study the consistency of several estimators which are based on different assumptions about variance structure of the error. Some decompositions of the finite population are introduced. Roughly speaking, we fit a weighted regression line to the finite population with the weight chosen according to the estimator under consideration. We show that any estimators in

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20. ABSTRACT - cont'd.

that class, except the ratio estimator, are inconsistent unless some conditions on the population's characteristics are satisfied. Based on the decomposition, modifications can be made to get a consistent estimator. For the case of p-auxiliary variables, we characterize the class of consistent weighted least squares estimators. The result is extended to the infinite population problem using a completely different approach.

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